#### P11 1.

Use Program 1.1 to approximate the fixed points (if any) of each function. Answers should be accurate to 12 decimal places. Produce a graph of each function and the line that clearly show any fixed points.

(c)   
additional Requirement: Produce a graph of function and the line that clearly shows any fixed points.

%plotfuc.m
  
figure;
  
x=-1.5:0.001:2;
  
g=x.^2-sin(x+15);
  
dg=2\*x-cos(x+0.15);
  
y=x;
  
plot(x,g,'LineWidth',1)
  
hold on
  
plot(x,y)
  
hold on
  
plot(x,dg)
  
hold on
  
plot([-1.5,2],[1,1],'--')
  
hold on
  
plot([-1.5,2],[-1,-1],'--')
  
hold on
  
line([-0.588,-0.588],[-4,5]);
  
hold on
  
line([0.825,0.825],[-4,5]);
  
legend('g(x)','y=x',"g'(x)")

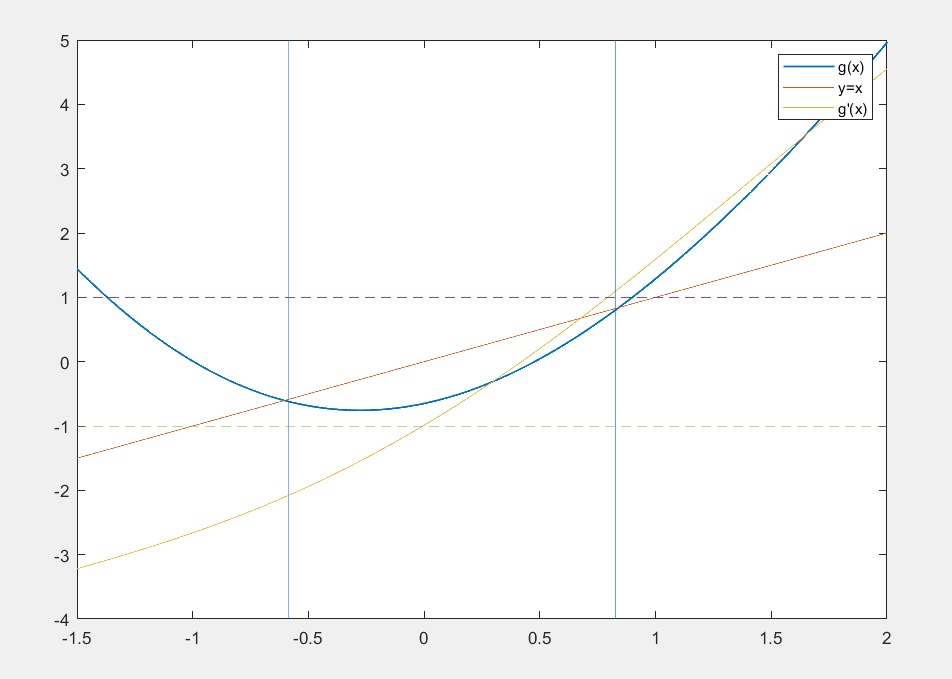


image-20210411142908482

We can see from the graph that has two fixed points within interval . But the derivatives of these two fixed points are greater than 1. Thus, they are repelling fixed points, we cannot use fixed-point iteration to find the solutions to the equation .

#### P22 1.

Find an approximation (accurate to 10 decimal places) for the interest rate that will yield a total annuity value of if monthly payments of are made.

**Solution:**  
Let be the monthly savings and be the annual interest rate .

|  |  |  |
| --- | --- | --- |
| Month | Principal | Money After Depositing |
|  |  |  |
| 2 |  |  |
| 3 |  |  |

After n month，the total amount of money is  
.  
Let ，.  
now, , , ，  
.

%moneyf.m
  
function [output] = moneyf(x)
  
 output=(300/(x/12))\*((1+(x/12))^240-1)\*(1+x/12);
  
end
  
   
%Command line window
  
>> moneyf(0.15)
  
ans =
  
 4.5479e+05
  
   
>> moneyf(0.16)
  
ans =
  
 5.2484e+05

Thus, the root of equation is within interval [0.15,0.16]. Use the False Position Method to find the solution of equation .

%f.m
  
function [output] = f(x)
  
output=(300/(x/12))\*((1+(x/12))^240-1)\*(1+x/12)-500000;
  
end
  
   
%regula.m
  
function [n,c,yc] = regula(f,a,b,delta,epsilon,max1)
  
digits(10)
  
ya=feval(f,a);
  
yb=feval(f,b);
  
if ya\*yb>0
  
 disp('Note:f(a)\*f(b)>0');
  
 return,
  
end
  
for k=1:max1
  
 dx=yb\*(b-a)/(yb-ya);
  
 c=b-dx;
  
 ac=c-a;
  
 yc=feval(f,c);
  
 if yc==0
  
 break;
  
 elseif yb\*yc>0
  
 b=c;
  
 yb=yc;
  
 else
  
 a=c;
  
 ya=yc;
  
 end
  
 dx=min(abs(dx),ac);
  
 if abs(dx)<delta,break,end
  
 if abs(yc)<epsilon,break,end
  
end
  
n=k;
  
c=vpa(c);
  
yc=feval(f,c);
  
   
%Command line window
  
>> [n,c,yc] = regula(@f,0.15,0.16,1e-10,1e-10,100)
  
n =
  
 6
  
   
c =
  
 0.1566
  
   
yc =
  
 -1.6076e-05

Thus , we find an approximation (accurate to 10 decimal places) for the interest rate that will yield a total annuity value of if monthly payments of are made.

#### P39 7.

Consider the function .  
**(a)** Find the Newton-Raphson formula .  
**(b)** If , then find and . What is ?  
**(c)** If , then find and . What is ?  
**(d)** What is the value of in part (c) ?

**Solution:**  
**(a)** The first derivate of is .  
The Newton-Raphson iterative function is .  
The Newton-Raphson formula is .

**(b)**

%f.m
  
function [output] = f(x)
  
output=x\*exp(-x);
  
end
  
   
%df.m
  
function [output] = df(x)
  
syms k
  
f(k)=k\*exp(-k);
  
df=diff(f(k));
  
k=x;
  
output=eval(df);
  
end
  
   
%newton.m
  
function [P,err,k,y] = newton(f,df,p0,delta,epsilon,max1)
  
digits(7)
  
P(1)=vpa(p0);
  
for k=1:max1
  
 p1=p0-feval(f,p0)/feval(df,p0);
  
 y(k)=vpa(feval(f,p0));
  
 err=abs(p1-p0);
  
 relerr=2\*err/(abs(p1)+delta);
  
 p0=p1;
  
 P(k+1)=vpa(p0);
  
 if(err<delta)|(relerr<delta)|(abs(y)<epsilon)
  
 break
  
 end
  
end

%Command line window  
>> [P,err,k,y] = newton(@f,@df,0.2,1e-12,1e-12,100)

The results are listed in table with starting value .

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 0 | 0.200000 | 0.163746 |
| 1 | -0.050000 | -0.052563 |
| 2 | -0.002380 | -0.002387 |
| 3 | -0.000006 | -0.000006 |
| 4 | -0.000000 | -0.000000 |

Thus, the fixed point is , .

**(c)-(d)**

%Command line window  
>> [P,err,k,y] = newton(@f,@df,2,1e-12,1e-12,100)

The results are listed in table with starting value .

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 0 | 2.000000 | 0.270671 |
| 1 | 4.000000 | 0.073263 |
| 2 | 5.333333 | 0.025749 |
| 3 | 6.564103 | 0.009256 |
| 4 | 7.743826 | 0.003356 |

，this time .